

## Confinement in large N gauge theories

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We report some recent results obtained for large N gauge theories which support the idea of volume reduction. Results for the string tension of the Twisted Eguchi-Kawai model match with those obtained from extrapolation from finite N. Determination of other observables is currently under way. Application of the twisted reduction idea to the case of 1 or 2 flavours of quarks in the adjoint representation of the group offers promising results. Preliminary results for the string tension point towards a very different behaviour for 1 and 2 flavours. While the string tension remains finite for 1 flavour at the critical massless quark limit, it seems to vanish with a large anomalous dimension for the  $N_f = 2$  case. This is consistent with the predicted Infrared fixed point expected in the latter case.

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## 1. Introduction

The idea of playing with the rank of the symmetry group as a parameter that can help to better understand the behaviour of a dynamical system, first appeared in the context of statistical mechanics. A large value for this rank suppresses fluctuations and the mean field approximation becomes increasingly accurate. 't Hooft[1] applied the same idea to the gauge group of a gauge invariant theory, but this time not all fluctuations are suppressed and the theory remains non-trivial. To date, the large  $N$  limit of  $SU(N)$  Yang-Mills theory remains unsolved. Nevertheless, there are several exact results that have been obtained (or conjectured) for this limit, which amount to a considerable simplification of the dynamics. For example, in perturbation theory only a subset of Feynman diagrams have to be considered at leading order in  $1/N$ . Furthermore, quark loops for a finite number of flavours are suppressed, and the quenched approximation is correct. This is only true if quarks are in the fundamental representation as for QCD. Fermionic fields in two-index representations (symmetric, antisymmetric or adjoint) are not suppressed, so that the large  $N$  dynamics is different in each case. There are also a large number of non-perturbative results that have been established or proposed.

Despite the time that has passed since the idea was first presented, the interest in large  $N$  gauge theories has not decreased. There are many hints that the dynamics in this limit is even quantitatively close to that of low rank theories as QCD. This makes it interesting at the phenomenological level. At a more conceptual level, the simplifications achieved at large  $N$  seem strong enough to hint that the theories might be, at least partially, solvable. These simplifications are not achieved at the cost of changing the basic dynamical properties of the theory: Confinement or chiral symmetry breaking etc are not lost at large  $N$ .

There is also a fascinating connection among ordinary quantum field theories and string theories which goes under the name *AdS/CFT*[2]. Again the large  $N$  limit implies an important simplification, leading to a classical description of the string side. As a matter of fact, the connection between the large  $N$  expansion and string theory is already suggested at the perturbative level: the  $1/N$  classification of Feynman diagrams follows the structure of the topological expansion of two-dimensional surfaces (the world sheet of a string).

Certain properties of large  $N$  theories adopt the form of conjectures, which rely on non-perturbative properties of the vacuum. Some of these are fairly recent, like the orientifold planar equivalence [3], which relates the dynamics of quarks in different types of two-index representations.

Testing these conjectures and extracting the spectra and other non-perturbative properties of large  $N$  gauge theories is clearly an important goal in quantum field theory. Unfortunately, the most powerful model-independent methodology —lattice gauge theories— does not seem to inherit the simplificatory character of the large colour limit. Indeed, the standard road to study these theories is by extrapolating the results of finite  $N$  gauge theories. A priori, the larger  $N$  the more computationally demanding is its study. Despite this fact, there have been many interesting studies indicating a fast approach to large  $N$ . For a recent review one can consult Ref. [4].

One of the earlier conjectures, known as *reduction*, seems to change this general pattern. The idea, put forward in Ref. [5], is that at large  $N$  results become volume independent. The derivation is done within the lattice approach and under the assumption that the  $Z(N)^d$  symmetry of the theory

is respected ( $d$  is the space-time dimension). Reduction has important conceptual and practical consequences, since the computational cost of higher  $N$  can be compensated by simulations done in smaller boxes. The extreme limit is that of a one-point lattice, and we end up with a matrix model of  $d$  matrices. The original proposal of Ref. [5], known as Eguchi-Kawai model, was soon disproved [6], since the assumption of  $Z(N)$  symmetry fails at weak coupling and  $d > 2$ , where the continuum limit is obtained. The authors of Ref. [6] proposed a modification called Quenched Eguchi-Kawai model which seemed to circumvent this problem, but has recently been shown to fail too.

## 2. The Twisted Eguchi-Kawai model

The present authors proposed a different modification called the Twisted Eguchi-Kawai model (TEK)[7], which also seemed to avoid the symmetry breaking problem. The idea is simple: formulate the finite volume lattice theory using twisted boundary conditions, and then reduce the model to a single point. The partition function is given by:

$$Z = \prod_{\mu} \int dU_{\mu} \exp\{-S_{\text{TEK}}(U_{\mu}, Z_{\mu\nu})\} \quad (2.1)$$

where the integral is over  $d$   $N \times N$  unitary matrices and the action is given by

$$S_{\text{TEK}}(U_{\mu}, Z_{\mu\nu}) = -b \sum_{\mu\nu} Z_{\mu\nu} \text{Tr}(U_{\mu} U_{\nu} U_{\mu}^{\dagger} U_{\nu}^{\dagger}) \quad (2.2)$$

The constants  $Z_{\mu\nu}$  are elements of the center and can be parameterised as  $\exp\{2\pi i n_{\mu\nu}/N\}$  in terms of an integer valued antisymmetric tensor  $n_{\mu\nu}$ . Different values for this tensor correspond to different boundary conditions. The particular choice  $n_{\mu\nu} = 0$  ( $Z_{\mu\nu} = 1$ ) corresponds to the original Eguchi-Kawai model. The weak coupling behaviour depends crucially on the value of  $n_{\mu\nu}$ . In four space-time dimensions, a particularly symmetric choice is achieved by taking  $N = L^2$  and  $n_{\mu\nu} = kL$  for  $\mu < \nu$ . In this case, at weak-coupling the  $Z(N)^4$  symmetry group is broken down to  $Z(L)^4$ , which is enough to preserve the reduction idea as  $L \rightarrow \infty$  ( $N \rightarrow \infty$ ).

The perturbative expansion of the TEK model gives rise to a Lie algebra expansion which mimicks the momentum expansion of a  $L^4$  lattice gauge theory. The propagator just coincides with that of an  $L^4$  volume, and the vertices contain momentum dependent phases which lead to rapidly oscillating factors for non-planar graphs. Indeed, the continuum version of the TEK model [8] produces a realization of non-commutative field theory [9]. This provides an explanation of the reduction idea at the perturbative level, which complements the Wilson loop equation of motion derivation of Eguchi and Kawai.

However, in the last few years there have been several results which challenge the validity of the TEK model. On one side, several numerical results [10]-[11]-[12] showed, that for large enough values of  $L$ ,  $Z(L)^4$  symmetry breaks down at intermediate values of the coupling  $b$  for the symmetric twist with minimal flux  $k = 1$ . On the other hand, instabilities in the perturbative expansion of non-commutative field theories were reported [13], which might transmit to instabilities in the  $Z(L)$ -symmetric vacuum of the TEK model [14].

In a previous paper [15], the present authors analyzed the problem and proposed that, in order to avoid symmetry breaking, the large  $N$  limit has to be taken by scaling  $k$  with  $L$  rather than keeping

$k$  fixed. In addition, to avoid instability problems of the type observed in non-commutative studies one should also keep  $\bar{k}/L$  finite, where  $\bar{k}$  is defined by  $k\bar{k} = 1 \bmod L$ . With these prescriptions incorporated, all numerical studies done at even higher values of  $L$  have shown no sign of symmetry breaking or instability.

The issue is far from being settled at the fundamental level. A full understanding of whether and how reduction takes place for the pure gauge theory is still missing. For practical reasons one should also understand the size and nature of the  $1/N$  corrections compared to the finite volume counterparts. A systematic study in this direction is hard, so that the authors decided to attack the simpler case of 2+1 dimensions first. A preliminary account of our findings has already appeared [16] and a full paper on the subject will soon follow. In that work we consider different fluxes  $k$ , different values of  $N$  and of the spatial size  $l$ . Perturbative calculations show that self-energy problems reported for non-commutative field theories are absent when the flux is scaled as proposed in Ref. [15].

### 3. New results for Pure gauge theory at large $N$

From a more practical standpoint it is much better to put the ideas to work in computing physical quantities of large  $N$  gauge theories with or without the use of reduction. For that purpose we set up a program to determine the string tension of the large  $N$  continuum Yang-Mills theory. Very nice previous analysis existed, of which we will single out that of Ref. [17] which used Polyakov loop correlation functions. For technical reasons we decided to use ordinary Ape-smeared Wilson loop expectation values. The goal was two-fold. On one hand we tried to develop a technique to extract precise values of the string tension from Creutz ratios of smeared Wilson loops. Once, these values were obtained for various  $N$ , the large  $N$  string tension value would be obtained by extrapolation. On the other hand, we would use the TEK model with our prescription of fluxes  $k$  to obtain what would tentatively be an alternative determination of the large  $N$  string tension.

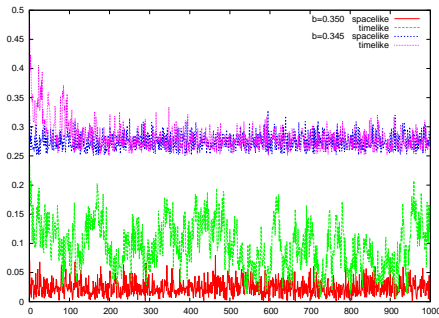
The main results of the previous analysis have been recently presented[18]. Results for groups  $SU(3)$ ,  $SU(4)$ ,  $SU(5)$ ,  $SU(6)$  and  $SU(8)$  fall nicely in the curve  $\frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = 0.515(3) + 0.34(1)/N^2$ . On the other hand the TEK model with  $N=841$  has  $\frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = 0.513(6)$ . Systematic errors entering in the definition of the scale are not included in the errors given, but do not affect the relative difference between the extrapolated and TEK result. Alternatively, one might determine the scale from the data itself, using a nonperturbative renormalization prescription. Once more, the conclusion is that the string tension for the TEK model matches the extrapolated one within the 2% error. This provides a numerical support for the validity of the reduction idea, implemented through the twisted one-site model.

Our work contains also interesting results concerning corrections to the area law behaviour of Wilson loops. This has implications for the effective string theory description of Wilson loops. It seems that the aspect ratio dependence of Wilson loops cannot be described entirely in terms of string fluctuations. One gluon exchange contributions turn out to be quite relevant. This is not surprising since a similar effective description including both singlet stringy components plus perturbative corrections has been found to be phenomenologically relevant since early times. Furthermore, there is a recent paper [19] in which, using a quite different but complementary analysis of Wilson loops at large  $N$ , the authors present results and conclusions in agreement with ours.

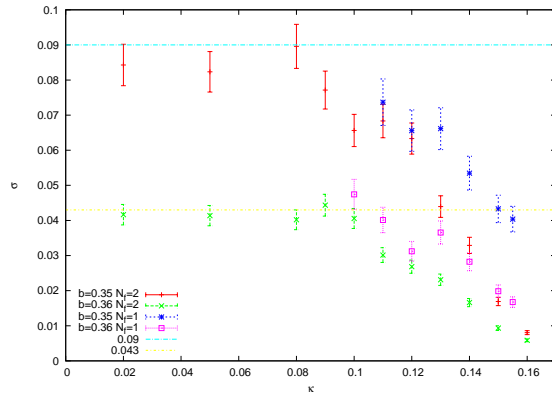
Up to now, the only physical quantities that have been analyzed with the use of the reduced model are the string tension and the asymptotic parameters mentioned before. However, a priori, the TEK model can be used to determine several other interesting quantities. We are currently exploring the determination of the chiral condensate, the finite temperature phase transition and the meson spectrum. At first, one might be puzzled by the possibility of determining these quantities with a single point matrix model. As an example, let us illustrate for example a possible program to determine the deconfinement temperature  $T_c$ .

The standard way to determine the deconfinement temperature is to study an asymmetric box  $L_0 \times L_s^3$  for  $L_0 \ll L_s$ . As the coupling  $b$  crosses a certain critical value  $b_c$ , the expectation value of time-like Polyakov loops ceases to vanish. The critical temperature is then given by  $T_c = 1/(L_0 a(b_c))$ , where  $a(b)$  is the lattice spacing in physical units. How can one obtain such a thing in a 1-point box? Indeed, there are two ways. One way is to modify the twist to make it asymmetric [20]. The other way, which we have explored, is to preserve the symmetric twist, but make the spatial  $b_s$  and time-like  $b_0$  couplings different. This generates also different scales  $a_s(b_0, b_s)$  and  $a_0(b_0, b_s)$ . Then a transition must occur as  $L a_0(b_0, b_s) = 1/T_c$  (where  $L = \sqrt{N}$ ). How would one notice the transition? We recall that we need vanishing Polyakov loops for reduction to operate. However, at weak coupling we know that the symmetry group breaks down to  $Z(L)^4$ . In the asymmetric coupling situation the deconfinement transition is signalled by the breaking from  $Z(N)$  to  $Z(L)$  in the temporal direction. The order parameter is then given by  $\text{Tr}(P_0)/N$ , where  $P_0 = U_0^L$  is the effective temporal Polyakov loop.

Does this happen in the reduced model with asymmetric couplings? The answer is yes, as exemplified in Fig. 1, describing the  $b_0/b_s = 4$  case. In it we show the Monte Carlo history of the modulus of  $\text{Tr}(U_\mu^L)/N$ . For smaller coupling, all expectation values are consistent with zero (shown by a small value of the modulus). This is given by the upper curves which are displaced by 0.25 to make them visible. A tiny change in  $b = \sqrt{b_0 b_s}$  then generates a situation displayed in the bottom curves. The spatial observables remain small and the temporal one becomes non-zero. Using this program one should be able to compute the critical temperature. The main difficulty is that now one has to use a more complex set of renormalization prescriptions to determine  $a_s(b_0, b_s)$  and  $a_0(b_0, b_s)$  separately.



(a) Fig. 1



(b) Fig. 2

#### 4. Adding $N_f$ flavours of quarks in the adjoint

The validity of reduction does not restrict to pure gauge theory. Indeed, on the contrary, it was proposed [21] that in theories with quarks in the adjoint representation the quark loops might help to restore the symmetry breaking even when using strictly periodic boundary conditions  $k = 0$ . The arguments are based on calculations done on  $S_1 \times R^3$ . The length of the periodic direction  $l_0$  becomes irrelevant in the large  $N$  limit, a phenomenon named *volume independence*. Indeed, for small  $l_0$  one can reliably compute the effective potential for the Polyakov loops. Gauge field contributions induce a breaking of  $Z(N)$  symmetry, while quark loops tend to restore it. This is strictly so for massless quarks. However, it has been claimed that symmetry restoration also applies in certain region with cut-off scale quark masses [22]. If this is so, quarks disappear from the low energy dynamics and one has simply an implementation of reduced pure gauge theory.

However, the gauge theory with dynamical quarks in the adjoint representation is interesting in its own right and has attracted considerable interest lately. Indeed, it is one of the candidates for a walking technicolor theory [23]. The gauge theory with  $N_f = 2$  flavours of quarks in the adjoint is expected to lie within the conformal window. Several studies have been carried for the  $SU(2)$  gauge theory on the lattice consistent with this fact (See for example Ref. [24]). The same is expected to hold for the large  $N$  counterpart. Furthermore, the large  $N$  gauge theory has many other attractive features. For example, via orientifold planar equivalence, the theory matches an alternative large  $N$  limit of QCD (two-index antisymmetric quark model). Furthermore, the case with a single Weyl fermion is supersymmetric.

Studying large  $N$  theory with dynamical fermions on the lattice poses a formidable computational challenge. It is quite clear that reduction can be crucial to obtain results at an affordable cost. For that reason, several authors have addressed the problem of simulating the corresponding reduced model for  $N_f = 1, 2$ , several values of  $N$  and different versions of lattice fermions. One or two flavours of overlap fermions have been studied by Hietanen and Narayanan [25]. Other authors [26]-[27] use Wilson fermions. In all cases they show evidence that  $Z(N)^4$  symmetry is preserved for a certain range of quark masses and values of  $N$ .

In the last few months we have addressed the problem from the perspective of the twisted reduced model. As a matter of fact, the fermionic part of the action is just that of a single site Wilson fermion coupled to the single site gauge field. The pure glue part is given by  $S_{\text{TEK}}$  as before. Several values of the integer flux parameter  $k$  were used, including  $k = 0$  corresponding to the untwisted case studied in Ref. [27]. Our study is still in progress, but a number of interesting conclusions have been obtained already. As expected, for appropriate ranges of  $k$ , there is no sign of symmetry breaking for arbitrary values of the quark mass and a range of  $N$  values going much beyond the one studied by previous authors. Furthermore, for large masses, the physical results obtained from Creutz ratios match with those obtained for the pure gauge theory. This confirms that, once twist is adopted, quark loops are an unnecessary ingredient in selecting the correct symmetry invariant vacuum.

A strong conclusion which we obtained from our work [28] is that the magnitude of the  $N$ -dependence of the results depends crucially on the value of  $k$ . For example, the plaquette expectation value for the untwisted  $k = 0$  case is strongly dependent on  $N$  with a pattern which is sometimes non-uniform. This casts serious doubts on the usefulness of the  $k = 0$  results to explore



$b$	$N_f$	$\kappa_c$	$\rho$	$\gamma^*$	$N_f$	$\kappa_c$	$\rho$
0.035	2	0.1694(7)	1.28(3)	1.00(5)	1	0.173(1)	1.25(3)
0.036	2	0.168(1)	1.33(3)	1.3(2)	1	0.171(1)	1.28(3)

the physics of the large  $N$  model. On the contrary, the results for large enough  $k$  values have quite small  $N$ -dependence. In the appropriate range of masses it seems, however, that the large  $N$  value of the plaquette is independent of  $k$ .

We have been able to obtain results up to  $N = 289$ . This corresponds to an effective box of length  $L = 17$ . This value is large enough to allow the study of Wilson loops up to  $8 \times 8$  size. A rough estimate of the string tension can be extracted using similar techniques as for the pure gauge case. In addition to smeared Wilson loops, we also studied the lowest eigenvalue  $\lambda$  of the square of the hermitian Wilson Dirac operator  $(\gamma_5 D_W)^2$ . The eigenvalue decreases with the hopping parameter  $\kappa$ , and seems to vanish at a critical value  $\kappa_c$ . Indeed, the data is well described by a formula

$$\propto \left( \frac{1}{\kappa} - \frac{1}{\kappa_c} \right)^\rho \quad (4.1)$$

The best fit parameters are summarized in the Table, for two values of  $b$  and  $N_f = 1, 2$ . We expect that the critical value signals the vanishing of quark masses.

In Fig. 2 we display the value of the string tension for  $b = 0.35, 0.36$  and  $N_f = 1, 2$ . For small  $\kappa$  (large bare masses) the string tension matches with the one corresponding to the pure gauge theory. The horizontal lines denote the value obtained from TEK and  $N = 841$ . At around  $\kappa \sim 1$  the string tension starts to depend strongly on  $\kappa$ . From the plot one sees an approximate linear behaviour. For  $N_f = 2$  the string tension seems to vanish around  $\kappa_c$ . A fit of the form Eq. 4.1 gives good  $\chi^2$  and  $\rho$  close to 1. Indeed setting  $\rho = 2/(1 + \gamma^*)$  the fit gives the values of  $\gamma^*$  plotted in the Table. The behaviour for  $N_f = 1$ , although also consistent with linearity, does not appear to vanish at  $\kappa_c$ .

## 5. Conclusions and outlook

In the previous sections we have studied various large  $N$  theories using the reduction idea. We have shown that computations are possible and, whenever the comparison could be made, match the results obtained by more conventional techniques. The numerical coincidence of the string tension implies that whatever mechanism drives Confinement it must be well described by our matrix model. As is well-known, the microscopic mechanism driving Confinement for pure Yang-Mills theory is not yet fully understood. This is so, despite the fact that we know since the early 70's that Confinement is a possible phase of gauge theories. To understand the meaning of microscopic mechanism, it is good to draw a parallelism with *superconductivity*. While condensation of a charged field is always at work, the BCS theory provides the microscopic mechanism that drives the condensation. For high temperature superconductors this theory is lacking.

In principle, the microscopic mechanism is specific of each theory and each dimensionality. We reject the common practice of exporting from one theory to other. This, however, does not apply to changes in  $N$ . As we saw, the string tension evolves mildly with  $N$ . Thus, there must also be a smooth continuity in the actual mechanism responsible for it, and a simple connection with the matrix model.

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